Section 12.6 Cylinders and Quadric Surfaces

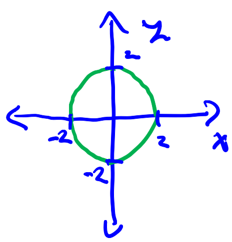
When sketching the graph of a surface, fix one of the variables and draw the corresponding *trace*. Traces or cross sections of the surfacec are the curves of intersection of the surface with planes parallel to the coordinate planes.

DEF: A *cylinder* is a surface that consists of all lines (called rulings) that are parallel to a given line and pass through a given plane curve.

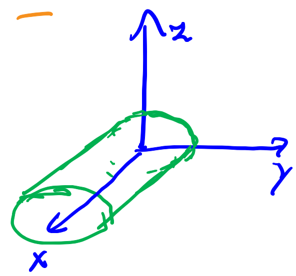
Ex1. Sketch the following cylinders:

(a) *Circular cylinder*: $y^2 + z^2 = 4$.

2D

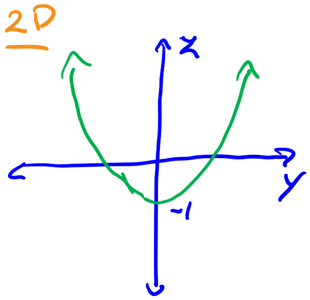


3D

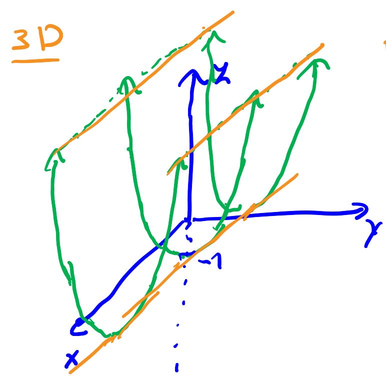


(b) *Parabolic cylinder*: $z = y^2 - 1$.

2D



3D

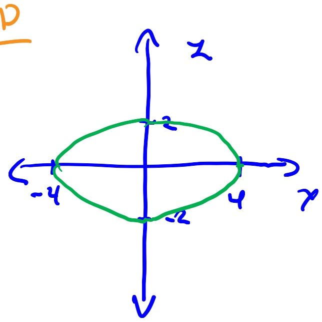


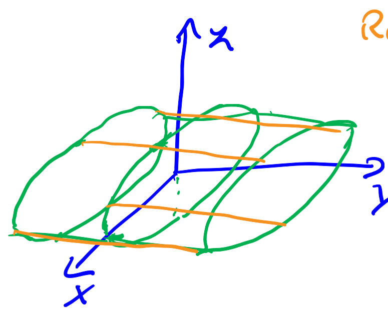
Rulings parallel to the x-axis

(c) *Elliptic cylinder*: $x^2 + 4z^2 = 16$.

$\to \frac{x^2}{16} + \frac{z^2}{4} = 1$

2D





Rulings are parallel to the y-axis

DEF: A *quadric surface* is the graph in space of a second-degree equation in x, y, and z.

Ex2. Use traces (or cross sections) to sketch the surface $z^2 - x^2 - y^2 = 1$

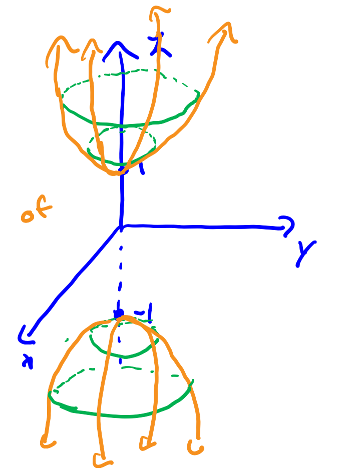
$z = k$: $k^2 - x^2 - y^2 = 1 \Rightarrow x^2 + y^2 = k^2 - 1$

- when $-1 < k < 1$, "nothing"
- when $k = 1$, $k = -1$, "a point"
- when $k > 1$, or $k < -1$, "circles"

$x = k$: $z^2 - k^2 - y^2 = 1 \Rightarrow z^2 - y^2 = 1 + k^2$ → Hyperbolas

$y = k$: $z^2 - x^2 - k^2 = 1 \Rightarrow z^2 - x^2 = 1 + k^2$

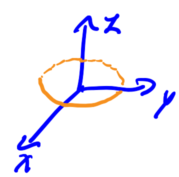
Hyperboloid of 2 sheets



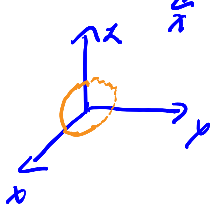
Ex3. Consider the surface $x^2 + \dfrac{y^2}{4} + z^2 = 1$, sketch the intersection of the surface and the planes listed below.

"ellipsoid"

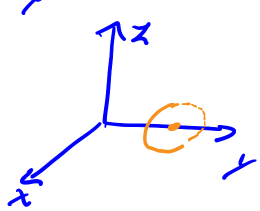
- Cross section at $z = 0$: $x^2 + \frac{y^2}{4} = 1$ "ellipse"



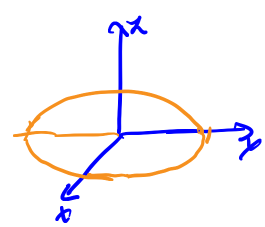
- Cross section at $y = 0$: $x^2 + z^2 = 1$ "circle"



- Cross section at $y = 1$: $x^2 + \frac{1}{4} + z^2 = 1$
  $\Rightarrow x^2 + z^2 = \frac{3}{4}$



- Cross section at $x = 0$: $\frac{y^2}{4} + z^2 = 1$



Standard Forms

Ellipsoid "fat ball"
$Ax^2 + By^2 + Cz^2 = 1$ $\qquad$ ($A > 0$, $B > 0$, and $C > 0$)

Hyperboloid of one Sheet
$Ax^2 + By^2 + Cz^2 = 1$ $\qquad$ (one of A, B, and C is negative and the other two are positive)

Hyperboloid of two Sheets
$Ax^2 + By^2 + Cz^2 = 1$ $\qquad$ (one of A, B, and C is positive and the other two are negative)

Elliptic Cone
$Ax^2 + By^2 + Cz^2 = 0$ zero $\qquad$ (one of A, B, and C is negative and the other two are positive)

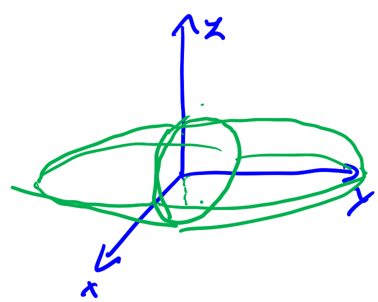
Elliptic Paraboloid
$z = Ax^2 + By^2$ "chip" $\qquad$ (A and B are either both negative or both positive)

Hyperbolic Paraboloid (Saddle)
$z = Ax^2 + By^2$ $\qquad$ (A and B have opposite signs)

Ex4. Identify the quadric surface and sketch.

(a) $x^2 + \frac{y^2}{4} + z^2 = 1$ $\qquad$ Ellipsoid

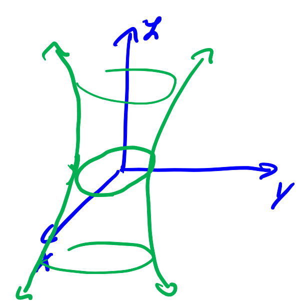
$+ \quad + \quad + \quad$ one



(b) $x^2 + y^2 - z^2 = 1$

$+ \quad + \quad - \quad$ one

Hyperboloid of one sheet



(c) $x^2 - y^2 + z^2 = 5 - 2y$

$x^2 - y^2 + 2y + z^2 = 5$

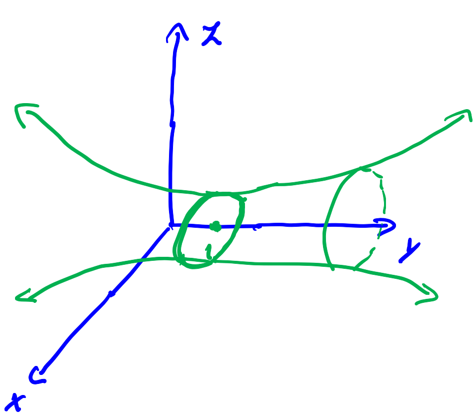
$x^2 - (y^2 - 2y - 1\ \ ) + z^2 = 5 + 1$

$x^2 - (y-1)^2 + z^2 = 4$

$\frac{x^2}{4} - \frac{(y-1)^2}{4} + \frac{z^2}{4} = 1$ one

$+ \quad - \quad +$

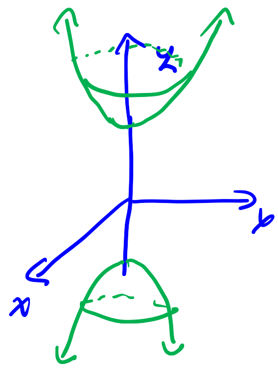
Hyperboloid of one sheet



(d) $z^2 - x^2 - y^2 = 1$

$+ \quad - \quad -$ ↑ one

Hyperboloid of two sheets

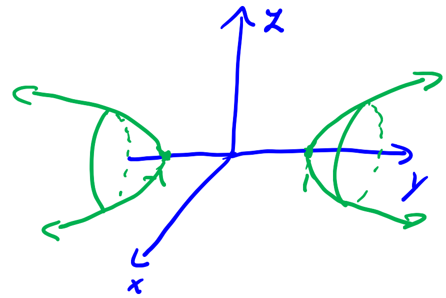


(e) $x^2 - y^2 + z^2 = -1$

$-x + y^2 - z^2 = 1$

$- \quad + \quad -$ ↑ one

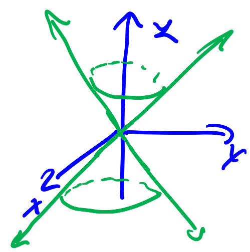
Hyperboloid of two sheets



(f) $x^2 + y^2 - z^2 = 0$

$+ \quad + \quad -$ ↑ zero!

Eliptic Cone



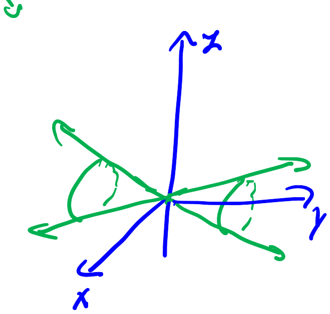
(g) $-3x^2 + y^2 - 3z^2 = 0$

↑ zero

$3x^2 - y^2 + 3z^2 = 0$

$+ \quad - \quad +$

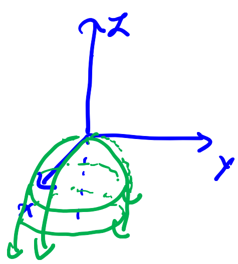
Eliptical Cone



(h) $z = -x^2 - y^2$

$- \quad -$

Eliptic paraboloid

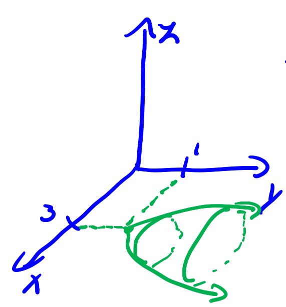


(i) $x^2 + 2z^2 - 6x - y + 10 = 0$

$x^2 - 6x + 9 + 2z^2 = y - 10$

$(x-3)^2 + 2z^2 = y - 1$

set $\quad X^2 + 2z^2 = y$



Eliptic Paraboloid

(j) $z = y^2 - x^2$

$+ \quad -$

Hyperbolic Paraboloid

geogebra

-3D online graphing tool